$\mathrm{AdS}_2 \times S^2$ as an exact heterotic string background

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ABSTRACT: An exact heterotic string theory on an $\mathrm{AdS}_2 \times S^2$ background is found as deformation of an $SL(2,\mathbb{R}) \times SU(2)$ WZW model.[†]

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1. Intro

Anti de Sitter in three dimensions and S^3 are among the most simple and yet interesting string backgrounds. They are exact solutions to the string equations beyond the supergravity approximation and, at the same time, are simple to deal with although non-trivial thanks to the presence of non-vanishing curvatures. For this reason they constitute an unique setting in which to analyze AdS/CFT correspondence, black-hole physics, little-string theory.

String propagation in these backgrounds is described in terms of WZW models for the $SL(2,\mathbb{R})$ and $SU(2)$ groups, hence marginal deformations of such models allow to study moduli space of the string vacua. In particular well-known class of marginal deformations for wzw models are those driven by left–right current bilinears [1, 2]. On the other hand S^3 and AdS_3 are embedded in larger structures so one can consider marginal deformations where just one of the currents belongs to the $SU(2)$ or AdS_3 algebra, the other belonging to some other $U(1)$ corresponding to an internal magnetic or electric field.

This kind of deformation generates a continuous line of exact CFT's. In this note we will show how with an appropriate choice for the deforming current we obtain a boundary in moduli space and that this boundary can be given a simple geometric interpretation [3, 4] in terms of the $\mathrm{AdS}_2 \times S^2$ near-horizon geometry of the Bertotti-Robinson black hole [5, 6].

2. $SU(2)$ asymmetric deformation

In the $SU(2)$ case, there exists just one possible choice for the deforming current the two other being related by inner automorphisms, since the group has rank one, is compact and its Lie algebra simple. Take the WZW model for $SU(2)$:

$$S_{SU(2)_k} = \frac{1}{2\pi} \int \mathrm{d}^2 z \left\{ \frac{k}{4} \left(\partial\alpha\bar{\partial}\alpha + \partial\beta\bar{\partial}\beta + \partial\gamma\bar{\partial}\gamma + 2\cos\beta\, \partial\alpha\bar{\partial}\gamma \right) + \sum_{a=1}^{3} \psi^a \bar{\partial}\psi^a \right\} \tag{2.1}$$

where ψ^a are the left-moving free fermions, superpartners of the bosonic $SU(2)_k$ currents, and (α, β, γ) are the usual Euler angles parameterizing the $SU(2)$ group manifold. The left-moving fermions transform in the adjoint of $SU(2)$; there are no right-moving superpartners but a right-moving current algebra of total charge $c = 16$ can be realized in terms of right-moving free fermions. This means that we can build a $\mathcal{N} = (1,0)$ world-sheet supersymmetry-compatible deformation given by:

$$\delta S_{\text{magnetic}} = \frac{\sqrt{k k_G} H}{2\pi} \int \mathrm{d}^2 z \left(J^3 + \imath \psi^1 \psi^2 \right) \bar{J}_G; \tag{2.2}$$

where J^3 belongs to the $SU(2)$ algebra and $\bar{J}_g$ is the current of the algebra at level k_g realized by the right-moving free fermions. An exact CFT is obtained for any value of the deformation parameter H.

2.1 Geometry

These new backgrounds all present a constant dilaton, a magnetic field, a NS-NS field proportional to the unperturbed one and a metric retaining a residual $SU(2) \times U(1)$ isometry [7]. The most remarkable property is that the deformation line in moduli space has a boundary corresponding to a critical value of the deformation parameter $H^2 = 1/2$. At this point the $U(1)$ subgroup decompactifies and the resulting geometry is the left coset $SU(2)/U(1) \sim S^2$ which is thus found to be an exact CFT background only supported by a magnetic field (the dilaton remains constant and NS field vanishes). A geometrical interpretation for this process can be given as follows: the initial S^3 sphere is a Hopf fibration of an S^1 fiber generated by the J^3 current over an S^2 base; the deformation only acts on the fiber, changing its radius up to the point where this seems to vanish, actually marking the trivialization of the fibration:

$$S^3 \xrightarrow[H^2 \to H^2_{\max}]{} \mathbb{R} \times S^2, \tag{2.3}$$

If we turn our attention to the gauge field one can show that a quantization of the magnetic charge is only compatible with levels of the affine algebras such that $\frac{k}{k_G} = p^2\ ,\ p \in \mathbb{Z}$. We will find the same condition in terms of the partition function for the boundary deformation.

Although this construction has been implicitly carried on for first order in α' background fields, it is important to stress that the resulting metric is nevertheless exact at all orders since the renormalization boils down to the redefinition of the level k that is simply shifted by the dual Coxeter number (just as in the WZW case).

2.2 Partition Function

Consider the case of $k_g = 2$ (one right-moving $\mathbb{C}$ fermion). The relevant components of the initial partition funciton are given by a $SU(2)_{k-2}$-modular-invariance-compatible combination of $SU(2)_{k-2}$ supersymmetric characters and fermions from the gauge sector. For our pourposes it is useful to further decompose the supersymmetric $SU(2)_k$ characters in terms of those of the $\mathcal{N} = 2$ minimal models:

$$\chi^j(\tau)\ \vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\tau,\nu) = \sum_{m \in \mathbb{Z}_{2k}} \mathcal{C}^j_m \begin{bmatrix} a \\ b \end{bmatrix} \Theta_{m,k}\left(\tau, -\frac{2\nu}{k}\right). \tag{2.4}$$

The deformation acts as a boost on the left-lattice contribution of the Cartan current of the supersymmetric $SU(2)_k$ and on the right current from the gauge sector:

$$\begin{aligned}
\Theta_{m,k}\ \bar\vartheta\begin{bmatrix} h \\ g \end{bmatrix} &= \sum_{n,\bar n} \mathrm{e}^{-\imath\pi g\left(\bar n + \frac{h}{2}\right)} q^{\frac{1}{2}\left(\sqrt{2k}n + \frac{m}{\sqrt{2k}}\right)^2} \bar q^{\frac{1}{2}\left(\bar n + \frac{h}{2}\right)^2} \\
&\longrightarrow \sum_{n,\bar n} \mathrm{e}^{-\imath\pi g\left(\bar n + \frac{h}{2}\right)}\ q^{\frac{1}{2}\left[\left(\sqrt{2k}n + \frac{m}{\sqrt{2k}}\right)\cosh x + \left(\bar n + \frac{h}{2}\right)\sinh x\right]^2} \\
&\qquad \times \bar q^{\frac{1}{2}\left[\left(\bar n + \frac{h}{2}\right)\cosh x + \left(\sqrt{2k}n + \frac{m}{\sqrt{2k}}\right)\sinh x\right]^2},
\end{aligned} \tag{2.5}$$

where the boost parameter x is given by $\cosh x = \frac{1}{1-2H^2}$.

Although an exact CFT is obtained for any value of the deformation parameter H we will concentrate, as before, on the boundary value $H^2 = 1/2$. In this case the boost parameter diverges thus giving the following constraints: $4(k+2)n+2m+2\sqrt{2k}\bar{n}+\sqrt{2k}h = 0$. Therefore, the limit is well-defined only if the level of the supersymmetric $SU(2)_k$ satisfies the quantization condition $k = 2p^2$, $p \in \mathbb{Z}$ *i.e.* the charge quantization for the flux of the gauge field. Under these constraints the $U(1)$ corresponding to the combination of charges orthogonal our condition decouples and can be removed. In this way we end up with the expression for the S^2 partition function:

$$Z_{S^2}\begin{bmatrix} a;h \\ b;g \end{bmatrix} = \sum_{j,\bar{\jmath}} M^{j\bar{\jmath}} \sum_{N\in\mathbb{Z}_{2p}} \mathrm{e}^{\imath\pi g\left(N+\frac{h}{2}\right)}\ \mathcal{C}^{j}_{p(2N-h)}\begin{bmatrix} a \\ b \end{bmatrix}\ \bar{\chi}^{\bar{\jmath}} \tag{2.6}$$

in agreement with the result found in [8] by using the coset construction. The remaining charge N labels the magnetic charge of the state under consideration.

3. $SL(2,R)$ deformation

The same construction as above can be repeated for the $SL(2,\mathbb{R})$ WZW model. In this case the moduli space is somewhat richer for it is possible to realize three different asymmetric deformations using the three generators of the group. These are not equivalent ($SL(2,\mathbb{R})$ is not compact) and in fact they lead to three physically different backgrounds. The elliptic deformation line, in example, contains the Gödel universe [9], the parabolic deformation gives the superposition of AdS_3 and a gravitational plane wave. Two of these deformation lines present the same boundary effect as the $SU(2)$ deformation. In particular the elliptic deformation leads to the hyperbolic space $\mathrm{H}_2 = SL(2,\mathbb{R})/U(1)$ supported by an immaginary magnetic field, *ie* an exact but non-unitary CFT. The hyperbolic deformation, on the other hand, leads to $\mathrm{AdS}_2 = SL(2,\mathbb{R})/U(1)$ supported by an electric field. No charge quantization is present in this case, because of the non-compact nature of the background.

In this latter case it is not yet possible to give the same construction for the partition function as for the $SU(2)$ case since this would require the decomposition of the initial partition function in a basis of hyperbolic characters which is not a simple exercize. Nevertheless by following the same procedure as before it is possible to evaluate the effect of the deformation on the spectrum of primaries and hence give the resulting AdS_2 background spectrum.

4. $\mathrm{AdS}_2 \times S^2$

The S^2 and AdS_2 backgrounds can be combined so to give an exact CFT corresponding to the $\mathrm{AdS}_2 \times S^2$ near-horizon geometry of the BR black-hole.

Let us now consider the complete heterotic string background which consists of the $\mathrm{AdS}_2 \times S^2$ space–time times an $\mathcal{N} = 2$ internal conformal field theory $\mathcal{M}$, that we will assume to be of central charge $\hat{c} = 6$ and with integral R-charges. The levels k of $SU(2)$

and $\hat{k}$ of $SL(2,\mathbb{R})$ are such that the string background is critical:

$$\hat{c} = \frac{2(k-2)}{k} + \frac{2(\hat{k}+2)}{\hat{k}} = 4 \implies k = \hat{k}. \tag{4.1}$$

This translates into the equality of the radii of the corresponding S^2 and AdS_2 factors, which is in turn necessary for supersymmetry. Furthermore, the charge quantization condition for the two-sphere restricts further the level to $k = 2p^2$, $p \in \mathbb{N}$.

The combined $\mathrm{AdS}_2 \times S^2$ background can give new insights about the physics of the BR black hole in particular by analizing the Schwinger-pair production in such background, or the study of the stability and propagation of D-branes.

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